Do Exchange Rates Convert Prices of Risk Across Countries?

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Abstract

Absence of arbitrage conditions impose important restrictions on the dynamics of bond and exchange rate returns. It can be shown that the exchange rate serves to convert prices of international undiversifiable risks from one currency to another. Put differently, arbitrage ensures that risk carries the same price in any two countries when evaluated from a particular viewpoint. As a consequence of this, expected returns should be equal after being converted to a common currency. We develop, estimate and test a linear 3-country asset pricing model for exchange risk hedged bond returns. Using US, UK, and German bond portfolio return data we find favorable evidence for the exchange rate being an unconditional converter of prices of risk across countries. Few other papers verify this important arbitrage pricing corollary.

Keywords: multi-country asset pricing model, exchange risk, price of risk conversion.
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1 Introduction

International arbitrage pricing theory (IAPT) imposes important restrictions on the joint dynamics of exchange rates and asset prices. In this paper it is shown and empirically validated that in order to exclude arbitrage opportunities exchange rates convert foreign prices of international risks to domestic prices of international risks. This implies that the expected excess return of an investor's international portfolio is determined by the factor prices of risk of the investor's home country. Put in other words, investors of a particular nationality can not change their domestic prices of risk since investing internationally entails exchange rate operations that effectively convert the foreign prices of risk into the domestic ones. This parity condition for the exchange rate in terms of excess returns is mentioned in most papers linking bond and exchange rate markets, e.g. Brandt and Santa-Clara (2001), Bansal (1995), Lund (1999) and Dewachter and Maes (2001).

To the best of our knowledge, this essential prediction of IAPT has not been the subject of extensive empirical research. Some researchers (see for instance Hardouvelis *et al.* (1999) or Cho *et al.* (1986)) have used it implicitly and partially to test for financial market integration. However, empirical analysis focussing on the price of risk conversion role of the exchange rate is lacking. In this paper, we focus exclusively on testing exactly this conversion property of exchange rates and its implications for excess returns from an unconditional point of view.

We propose a linear orthogonal factor model for local currency[1] bond portfolio returns along the lines of Solnik (1983). However, his IAPT model states that exchange rate returns are fully spanned by bond return factors, which is an assumption we find to be counterintuitive and at odds with the data. Indeed, exchange rate returns are only partially spanned by local currency bond return factors. Ikeda (1991) acknowledges this and puts forward a different IAPT model in which a significant exchange rate risk component is orthogonal to the bond return factors. Ikeda explains arbitrage pricing of hedged securities in terms of local factor risk premia. More in particular, he claims that expected excess bond returns, after being hedged against exchange risk, are spanned by international factor loadings with local currency market prices of risks serving as weights. As far as we known, this model has not been tested empirically and this despite of its intuitive appeal. In this paper we fill this gap in the literature and analyze whether or not the model claims are validated using US, UK, and German bond portfolio data.

We statistically filter the international common risk factors from bond returns by applying a principal factor analysis on the correlation matrix of the local currency returns (see also Driessen, Melenberg, and Nijman (2000)). Next, we employ the generalized method of moments of Hansen (1982) to infer the currency dependent market prices of risk. Finally, we test the model claims using a standard GMM overidentifying restrictions test. Interestingly,

[1] *Local currency* returns are defined as the returns denominated in the currency of the geographical place where the investments are quoted. In that sense, a domestic investor does not measure (or care about) the local currency returns of his foreign investments, but rather evaluates the returns of his international investments expressed in his own domestic currency.

we find evidence in favor of his model, and as such in favor of the exchange rate being a price of risk converter across countries.

The structure of the rest of this paper is as follows. In section 2 we rephrase Ikeda's (1991) model for hedged bond returns in a multi-country continuous time setting. In section 3 we propose unconditional overidentifying restrictions tests for the absence of arbitrage relationships. Section 4 constructs and discusses the data, presents estimation and test results for a 3-country arbitrage pricing model, and briefly discusses an application. Finally, we present conclusions in section 5.

2 Theoretical framework

2.1 A linear factor model with exchange risk

We extend the standard APT model of Ross (1976) to an international setting where the exchange rate is not necessarily spanned by the bond market risk factors alone. We basically repeat the reasoning of Ikeda (1991) in a continuous time framework and allowing for more than one risky asset in each country. We consider a world with M countries (and corresponding M currencies) indexed 1 to M. In each country, there exist N risky assets and 1 locally riskless national bond. We assume that these assets are freely traded in perfect international capital markets. Denote the price of the risky asset n of country m, denominated in (local) currency m, as P_n^m, $n = \{1, ..., N\}$, $m = \{1, ..., M\}$. These asset prices are supposed to be driven by K international (undiversifiable) risks and 1 national (diversifiable) aggregated risk component. The K international factors are represented by Wiener processes W_k, $k = \{1, ..., K\}$, while the components that are idiosyncratic to P_n^m are denoted by Z_n^m. The dynamics of the local currency (currency-m denominated) prices P_n^m is supposed to be given by the following equation, for all m and n:

$$\frac{dP_n^m}{P_n^m} = \mu_n^m dt + \sum_{k=1}^{K} L_{nk}^m dW_k + dZ_n^m. \tag{1}$$

In the equation above, dP_n^m/P_n^m is the random return on the m-th country n-th risky asset in terms of the local currency m, $\mu_n^m dt$ denotes its expected instantaneous value, dW_k represents the kth zero mean international risk factor, L_{nk}^m denotes the sensitivity of the return dP_n^m/P_n^m to fluctuations in the factor dW_k, and dZ_n^m is a non-systematic risk component with zero mean, bounded variance, and $E\left[dZ_n^m \middle| dW_k\right] = 0$, for all n, m, and k. The number of risky assets, NM, is assumed to be large enough such that the law of large numbers may safely be assumed to hold.

At the end of the day, investors that invest across countries do not care about returns in local currency[2] since they face exchange rate changes when evaluating the proceeds of their international investment strategy. We define the exchange rate $S_{m^*}^m$ as the unit price of foreign

[2] We repeat that by *local currency* we mean the currency of the country in which the asset is originally quoted (*i.e.* the country of the investment and not necessarily the country of the investor).

currency m^* in terms of domestic currency m, hence $dS^m_{m^*}/S^m_{m^*}$ is the rate of appreciation of currency m^* in terms of currency m, for all m and m^*. We assume flexible foreign exchange rates and denote their random returns as:

$$\frac{dS^m_{m^*}}{S^m_{m^*}} = \mu^{S^m_{m^*}} dt + \sigma^{S^m_{m^*}}. \tag{2}$$

Values $\mu^{S^m_{m^*}} dt$ and $\sigma^{S^m_{m^*}}$ denote the expected and random instantaneous parts of exchange rate variation respectively. We leave them unspecified for the moment. The price of asset n in country m^* measured by a country m-investor is given by the law of one price as $\left(P^{m^*}_n\right)^m = P^{m^*}_n S^m_{m^*}$. From this, an application of Itô's lemma yields:

$$\left(\frac{dP^{m^*}_n}{P^{m^*}_n}\right)^m = \frac{dP^{m^*}_n}{P^{m^*}_n} + \frac{dS^m_{m^*}}{S^m_{m^*}} + \frac{dP^{m^*}_n}{P^{m^*}_n}\frac{dS^m_{m^*}}{S^m_{m^*}}. \tag{3}$$

Substitution of (1) and (2) into (3) yields:

$$\left(\frac{dP^{m^*}_n}{P^{m^*}_n}\right)^m = \left(\mu^{m^*}_n\right)^m dt + \sum_{k=1}^{K} L^{m^*}_{nk} dW_k + dZ^{m^*}_n + \sigma^{S^m_{m^*}}, \tag{4}$$

where we pooled some terms into $\left(\mu^{m^*}_n\right)^m dt$:

$$\left(\mu^{m^*}_n\right)^m dt = \mu^{m^*}_n dt + \mu^{S^m_{m^*}} dt + \frac{dP^{m^*}_n}{P^{m^*}_n}\frac{dS^m_{m^*}}{S^m_{m^*}}. \tag{5}$$

The term $\sigma^{S^m_{m^*}}$ in (4) is of crucial importance since it impedes the construction of a riskless portfolio as will be shown now. Construct a portfolio from the NM risky assets, denoted by $\mathbf{w} = (w_1, ..., w_{NM})'$ with $\mathbf{w}$ being the investment proportions in the assets that are ranked as follows: $P^1_1, ..., P^1_N, P^2_1, ..., P^2_N, ..., P^M_1, ..., P^M_N$. According to the usual APT rule, $\mathbf{w}$ is chosen such that:

$$\begin{cases} \mathbf{w}'\mathbf{L}_1 = 0 \\ ... \\ \mathbf{w}'\mathbf{L}_K = 0 \end{cases} \tag{6}$$

$$\begin{aligned} &\mathbf{w}'\boldsymbol{\iota}_{NM} = 1, \\ &\mathbf{w}'\mathbf{dZ} \simeq 0, \end{aligned} \tag{7}$$

where $\mathbf{L}_k = \left(L^1_{1k}, ..., L^1_{Nk}, ..., L^M_{1k}, ..., L^M_{Nk}\right)'$, $\mathbf{dZ} = \left(dZ^1_1, ..., dZ^1_N, ..., dZ^M_1, ..., dZ^M_N\right)'$, $\boldsymbol{\iota}_{NM}$ a $NM \times 1$ vector of ones, and where the last approximate equality follows from the assumption that dZ^m_n is idiosyncratic (or non-systematic) for all n and m. The currency-m return on the portfolio $\mathbf{w}$, $\left(\frac{dP}{P}\right)^m_w$, can be computed from equations (4), (6) and (7) as:

$$\begin{aligned} \left(\frac{dP}{P}\right)^m_w = \mathbf{w}'\left(\frac{d\mathbf{P}}{\mathbf{P}}\right)^m &= \mathbf{w}'\boldsymbol{\mu}^m dt + \sum_{k=1}^{K}\mathbf{w}'\mathbf{L}_k dW_k + \mathbf{w}'\mathbf{dZ} + \mathbf{w}'\boldsymbol{\sigma}^{S^m} \\ &= \mathbf{w}'\boldsymbol{\mu}^m dt + \mathbf{w}'\boldsymbol{\sigma}^{S^m} \end{aligned} \tag{8}$$

where

$$
\begin{aligned}
(d\mathbf{P}/\mathbf{P})^m &= \left(\left(dP_1^1/P_1^1\right)^m, ..., \left(dP_N^1/P_N^1\right)^m, ..., \left(dP_1^M/P_1^M\right)^m, ..., \left(dP_N^M/P_N^M\right)^m\right)', \\
\boldsymbol{\mu}^m &= \left(\left(\mu_1^1\right)^m, ..., \left(\mu_N^1\right)^m, ..., \left(\mu_1^M\right)^m, ..., \left(\mu_N^M\right)^m\right)' \\
\boldsymbol{\sigma}^{S^m} &= \left(\boldsymbol{\iota}_N \otimes \sigma^{S_1^m}, ..., \boldsymbol{\iota}_N \otimes \sigma^{S_M^m}\right)
\end{aligned}
$$

The equation (8) reveals that the simple hedging rule of the APT does not yield a locally risk free portfolio since the exchange risk would be left undiversified by this rule. The next subsection confronts and solves this problem by considering an arbitrage portfolio which hedges against exchange risk by means of foreign lending and borrowing at locally riskless rates.

2.2 Arbitrage pricing of hedged assets

There are basically two equivalent ways of hedging the exchange risk when investing abroad: *(i)* go to the forward market and enter a contract to deliver an amount of foreign currency that is equivalent to its investment at a given date, or *(ii)* borrow foreign currency on the foreign market and sell it immediately (*i.e.* exchange it for your own currency at the spot exchange rate). In turn you can invest it in your own locally riskless bond B^m (where m denotes your country). While the forward market is typically used for short term investment protection or speculation, the second method is more adapted to portfolio investment. We employ the second approach and go short in (borrow) the risk free asset of the foreign country B^{m^*}.

Consider a portfolio of $2NM$ assets, $\boldsymbol{\theta} = (\theta_1, ..., \theta_{2NM})'$, where proportion θ_i is invested in risky asset i (for ranking see previous section), and where proportion θ_{NM+i}, $i = \{1, ..., NM\}$, is invested in the national risk free asset of the country under consideration.[3] Based on the portfolio $\mathbf{w}$ as defined in (6), we now specify portfolio $\boldsymbol{\theta}$ as:

$$
\begin{aligned}
&\theta_i = w_i \qquad\qquad (i = 1, ..., NM) \\
&\theta_{NM+i} = -w_i \qquad\qquad (i = 1, ..., NM).
\end{aligned}
\tag{9}
$$

The resulting bundle is a riskless arbitrage portfolio. The risk originating from both international (dW) and residual factors (dZ) is diversified and one is now protected against the exchange risk in the risky asset by opposite trading in the national bond of the foreign country. Explicitly, if $\left(dB^{m^*}/B^{m^*}\right)^m$ denotes the currency-m return on the deterministic national

[3] Note that each of the groups of weights $\theta_{NM+1}, ..., \theta_{NM+N}$ up until $\theta_{NM+(NM-N+1)}, ..., \theta_{NM+NM}$ apply to the risk free asset of country 1 up until M respectively. The reason why we do not pool per country is for convenience and will become clear in the following.

bank account of country m^*, for all m^*:

$$\begin{aligned}
\left(\frac{dB^{m^*}}{B^{m^*}}\right)^m &= \frac{dB^{m^*}}{B^{m^*}} + \frac{dS^m_{m^*}}{S^m_{m^*}} + 0, \\
\left(\frac{dB^{m^*}}{B^{m^*}}\right)^m &= \frac{dB^{m^*}}{B^{m^*}} + \mu^{S^m_{m^*}} dt + \sigma^{S^m_{m^*}}.
\end{aligned} \tag{10}$$

Hence, combining (8), (9), and (10), the currency m-return on portfolio $\boldsymbol{\theta}$, $\left(\frac{dP}{P}\right)^m_\theta$, reduces to:

$$\begin{aligned}
\left(\frac{dP}{P}\right)^m_\theta &= \mathbf{w}'\left(\frac{\mathbf{dP}}{\mathbf{P}}\right)^m - \mathbf{w}'\left(\frac{\mathbf{dB}}{\mathbf{B}}\right)^m \\
&= \mathbf{w}'\boldsymbol{\mu}^m dt + \mathbf{w}'\boldsymbol{\sigma}^{S^m} - \mathbf{w}'\frac{\mathbf{dB}}{\mathbf{B}} - \mathbf{w}'\boldsymbol{\mu}^{S^m} dt - \mathbf{w}'\boldsymbol{\sigma}^{S^m} \\
&= \mathbf{w}'\boldsymbol{\mu}^m dt - \mathbf{w}'\frac{\mathbf{dB}}{\mathbf{B}} - \mathbf{w}'\boldsymbol{\mu}^{S^m} dt,
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
(\mathbf{dB}/\mathbf{B})^m &= \left(\boldsymbol{\iota}_N \otimes \left(dB^1/B^1\right)^m, ..., \boldsymbol{\iota}_N \otimes \left(dB^M/B^M\right)^m\right) \\
\mathbf{dB}/\mathbf{B} &= \left(\boldsymbol{\iota}_N \otimes dB^1/B^1, ..., \boldsymbol{\iota}_N \otimes dB^M/B^M\right) \\
\boldsymbol{\mu}^{S^m} &= \left(\boldsymbol{\iota}_N \otimes \mu^{S^m_1}, ..., \boldsymbol{\iota}_N \otimes \mu^{S^m_M}\right)
\end{aligned}$$

According to equation (11) the return measured in currency-m is indeed free from any risk[4]. Now that a riskless arbitrage portfolio has been obtained from the viewpoint of currency m, we can use the same algebraic argument as the standard APT to determine risk premia for country-m's investors. For free lunches to be absent, the currency m return on portfolio θ must be zero, or:

$$\begin{aligned}
\left(\frac{dP}{P}\right)^m_\theta &= 0, \\
\mathbf{w}'\left(\boldsymbol{\mu}^m dt - \frac{\mathbf{dB}}{\mathbf{B}} - \boldsymbol{\mu}^{S^m} dt\right) &= 0.
\end{aligned} \tag{12}$$

From equation (6) it can be seen that the vector $\mathbf{w}$ is orthogonal to each of the loadings. From the equation above we see that this vector is also orthogonal to the vector of expected excess hedged returns, hence linear algebra says that this vector of expected excess hedged returns should be a linear combination of the loading vectors. Or, $\boldsymbol{\mu}^m dt - \frac{\mathbf{dB}}{\mathbf{B}} - \boldsymbol{\mu}^{S^m} dt$ must be spanned by factor loading vectors $\mathbf{L}_k$ ($k = \{1, ..., K\}$) for some parameters $\lambda^m_1, ..., \lambda^m_K$. This leads to Ikeda's main result, namely that expected net returns, in terms of currency m and

[4] Note that even though exchange rate changes have been hedged, exchange rate dynamics still play a role in the determination of the excess holding returns in local currency through the third component of $\mu^{m_2} dt$ (see equation (5)). As explained in Eun and Resnick (1988), the reason is that foreign currency asset returns contain stochastic, unpredictable components which by definition can not be hedged. These unexpected returns have to be converted into local currency returns on the spot market resulting in the covariance term. Thus, to the extent that the exchange rate stochastics depend on the international bond return factors, this covariance term differs from zero.

on risky assets whose exchange risk is hedged by foreign borrowing and lending, are spanned by common factor loading vectors $\mathbf{L}_k$ with weights $\lambda_1^m, ..., \lambda_K^m$, or:

$$\boldsymbol{\mu}^m dt - \frac{\mathbf{dB}}{\mathbf{B}} - \boldsymbol{\mu}^{S^m} dt = \lambda_1^m \mathbf{L}_1 + ... + \lambda_K^m \mathbf{L}_K. \tag{13}$$

In words, in each currency m there are K prices of risks which determine the expected returns on hedged assets. What is important to note is that the hedged returns will be determined by the prices of risk of their currency of denomination. Apparently, a currency-m investor will not be able to apply the prices of risk of the country where he is investing to determine the expected return from investing abroad, if no-arbitrage is to hold.

2.3 Arbitrage determination of asset-currency covariances

In this section we will motivate why the exchange rate converts prices of risks across countries. Consider an investment made by a country-m investor in the nth asset of country m^*, $P_n^{m^*}$. Obviously, from equation (13), the investor expects the following currency-m hedged return for this investment:

$$\left(\mu_n^{m^*}\right)^m dt - \frac{dB^m}{B^m} - \mu^{S^m_{m^*}} dt = \lambda_1^m L_{n1}^{m^*} + ... + \lambda_K^m L_{nK}^{m^*}. \tag{14}$$

Next, substitute (5):

$$\mu_n^{m^*} dt + \frac{dP_n^{m^*}}{P_n^{m^*}} \frac{dS_{m^*}^m}{S_{m^*}^m} - \frac{dB^m}{B^m} = \lambda_1^m L_{n1}^{m^*} + ... + \lambda_K^m L_{nK}^{m^*}. \tag{15}$$

From the viewpoint of a country-m^* investor (replace m by m^* in the above), the equation may be rewritten as:

$$\mu_n^{m^*} dt - \frac{dB^{m^*}}{B^{m^*}} = \lambda_1^{m^*} L_{n1}^{m^*} + ... + \lambda_K^{m^*} L_{nK}^{m^*}. \tag{16}$$

Comparing (16) with (15), we see that the presence of the cross term $\frac{dP_n^{m^*}}{P_n^{m^*}} \frac{dS_{m^*}^m}{S_{m^*}^m}$ in (15) effectively converts the set of prices of risk from $\lambda_k^{m^*}$ to λ_k^m ($k = \{1, ..., K\}$). By taking the difference between equations (15) and (16), the covariance term between asset returns and exchange rate returns may be written as:

$$\frac{dP_n^{m^*}}{P_n^{m^*}} \frac{dS_{m^*}^m}{S_{m^*}^m} = \left(\lambda_1^m - \lambda_1^{m^*}\right) L_{n1}^{m^*} + ... + \left(\lambda_K^m - \lambda_K^{m^*}\right) L_{nK}^{m^*}. \tag{17}$$

Hence we see that the covariances between asset returns and currency returns are also spanned by factor loadings $\mathbf{L}_k$. But here the weights are risk price differences $\left(\lambda_k^m - \lambda_k^{m^*}\right)$. Intuitively put, the above result means that the exchange rate serves at least to one purpose, that is, to convert the prices of risk of foreign investments to the ones of the investor's home country. Therefore, an investor (denominating foreign investment proceeds in local currency) will never be able to change the prices of risk from those that apply in his home market. This is an especially strong result extending the standard intuition of the uncovered interest rate condition to a risk averse world. Likewise we have the analogy for the foreign investor. He will also end up with the prices of risk in the foreign market (once he starts denominating his investment proceeds in his own currency, that is the foreign currency).

If we specify $\sigma^{S^m_{m^*}}$ in terms of the international bond market factors and an idiosyncratic exchange rate factor $dZ^{S^m_{m^*}}$, equation (2) may be written as:

$$\frac{dS^m_{m^*}}{S^m_{m^*}} = \mu^{S^m_{m^*}} dt + \sum_{k=1}^{K} \Gamma^m_{m^*,k} dW_k + dZ^{S^m_{m^*}}, \tag{18}$$

where $\Gamma^m_{m^*,k}$ is a scalar for each k. Hence we can rewrite the left hand side of equation (17) as:

$$\sum_{k=1}^{K} L^{m^*}_{nk} \Gamma^m_{m^*,k} = \sum_{k=1}^{K} \left(\lambda^m_k - \lambda^{m^*}_k\right) L^{m^*}_{nk}, \tag{19}$$

which holds if:

$$\Gamma^m_{m^*,k} = \left(\lambda^m_k - \lambda^{m^*}_k\right) \qquad \text{for all } k. \tag{20}$$

The implications for expected returns follow immediately. First, as long as exchange rate risks are hedged (*ex ante*), expected excess holding returns only depend on local currency prices of factor risks. Otherwise stated, domestic and foreign portfolios with identical factor risk profiles will result in identical expected excess holding returns, once converted into a common currency. Second, this result does not need to hold for non-hedged portfolios. There, exchange rate dynamics and factors enter fully. As such, excess holding returns will partly be determined by the forward premium, *i.e.* the price of holding additional and orthogonal exchange rate risks. Unconditionally, however, there is evidence that deviations from uncovered interest rate parity are small, implying negligible forward premia. In this sense, one may expect the expected returns to equalize as well for unhedged portfolios.

3 Unconditional tests for arbitrage opportunities

Equation (13) suggests a test for the conversion property of the exchange rate: the size of the expected excess holding return on any hedged assets is determined by the factor loadings on the international risks evaluated at the currency-specific market prices of risk, *i.e.* the market prices of risk of the investor's home market. Basically, we will construct the discrete-time empirical counterpart of equation (13) and consider these as moments to be fitted by the theoretical model as set out in the previous section. For convenience, we repeat equation (13) above:

$$\boldsymbol{\mu}^m dt - \frac{\mathbf{dB}}{\mathbf{B}} - \boldsymbol{\mu}^{S^m} dt = \mathbf{L}\boldsymbol{\lambda}^m \text{ for all } m = \{1, ..., M\}. \tag{21}$$

where $\boldsymbol{\lambda}^m$ denotes the $K \times 1$ vector of prices of risk in currency denomination of country m, and where $\mathbf{L} = (\mathbf{L}_1, ..., \mathbf{L}_K)$ is the $NM \times K$ matrix that stacks the loading vectors of each international factor. Per viewpoint, we have NM assets to consider. Denote the $NM^2 \times 1$ vector stacking the exchange risk hedged expected excess returns from the M different points of view by $\boldsymbol{\mu}^F$ and analogously, define an $MK \times 1$ vector stacking the factor prices of risk by $\boldsymbol{\lambda}^F$ then all relevant arbitrage conditions of equation (21) can be summarized as:

$$\boldsymbol{\mu}^F = (\mathbf{I}_M \otimes \mathbf{L})\boldsymbol{\lambda}^F. \tag{22}$$

Equation (22) implies a number of cross-sectional restrictions on the expected excess returns. More in particular, since there are but MK (unknown) prices of risk in NM^2 expected excess holding returns we have $M(NM - K)$ cross-sectional restrictions on excess returns that can be tested statistically. In the following empirical section we choose for a GMM based test statistic. More specifically, we perform a principal factor analysis to obtain the respective local currency factors and loadings, $\mathbf{F}_t$ and $\mathbf{L}$ (more details can be found in the appendix). In a second stage, we estimate the prices of risk vector $\boldsymbol{\lambda}^F$. More specifically, we construct the empirical counterpart, $\boldsymbol{\mu}^{emp}$, of the $NM^2 \times 1$ vector $\boldsymbol{\mu}^F$, and minimize the sum of squared errors with respect to $\boldsymbol{\lambda}^F$:

$$\boldsymbol{\lambda}^F = \arg\min((\boldsymbol{\mu}^{emp} - (\mathbf{I}_M \otimes \mathbf{L})\boldsymbol{\lambda}^F)'\mathbf{W}(\boldsymbol{\mu}^{emp} - (\mathbf{I}_M \otimes \mathbf{L})\boldsymbol{\lambda}^F)). \tag{23}$$

Because of the linearity we have that the estimate of $\boldsymbol{\lambda}^F$ is known in closed form. Assuming an identity matrix for $\mathbf{W}$, we have:

$$\hat{\boldsymbol{\lambda}}^F = (\,(\mathbf{I}_M \otimes \mathbf{L})'(\mathbf{I}_M \otimes \mathbf{L}))^{-1}(\mathbf{I}_M \otimes \mathbf{L})'\boldsymbol{\mu}^{emp}. \tag{24}$$

Note that the above equation hints at getting the respective currency specific prices of risk from M separate cross-sectional regressions, regressing the average excess returns on the respective factor loadings obtained from the factor analysis. From this procedure we obtain the GMM test statistics for cross-sectional restrictions. We use these prices of risk and pricing errors to test for correct price of risk conversion. Under the null hypothesis that the model is a valid representation of the excess returns we have that Hansen's J-statistic is $\chi^2\left(M\left(MN-K\right)\right)$ distributed, *i.e.*:

$$TJ_T^1 = T((\boldsymbol{\mu}^{emp} - (\mathbf{I}_M \otimes \mathbf{L})\boldsymbol{\lambda}^F)'\boldsymbol{\Xi}^{-1}(\boldsymbol{\mu}^{emp} - (\mathbf{I}_M \otimes \mathbf{L})\boldsymbol{\lambda}^F)) \sim \chi^2\left(M\left(MN-K\right)\right), \tag{25}$$

where T denotes the time series dimension, *i.e.* the number of observed returns per series, and $\boldsymbol{\Xi}$ is a consistent estimate of the covariance matrix of the sample pricing errors (see e.g. Cochrane (1996)). We also employ this procedure to test for the conversion role played by the exchange rate. For instance, if exchange rates do not covary with excess holding returns of foreign assets, the exchange rate is but a nuisance term in international finance and according to the IAPT, prices of risk will be equal across countries (currencies). Testing for the equality of prices of risk across countries can be preformed by imposing the null hypothesis $\boldsymbol{\lambda}^F = \boldsymbol{\iota}_M \otimes \boldsymbol{\lambda}$, where $\boldsymbol{\lambda} \equiv \boldsymbol{\lambda}^1 = ... = \boldsymbol{\lambda}^M$, resulting in the test statistic:

$$TJ_T^2 = T((\boldsymbol{\mu}^{emp} - (\mathbf{I}_M \otimes \mathbf{L})\left(\boldsymbol{\iota}_M \otimes \boldsymbol{\lambda}\right))'\boldsymbol{\Xi}^{-1}(\boldsymbol{\mu}^{emp} - (\mathbf{I}_M \otimes \mathbf{L})\left(\boldsymbol{\iota}_M \otimes \boldsymbol{\lambda}\right))) \sim \chi^2\left(M^2N-K\right) \tag{26}$$

Equations (25) and (26) constitute the core of this paper and are used in the next section to analyze the role of the exchange rate. First, equation (26) allows us to analyze to what extent countries differ in pricing international risk factors. If prices differ across countries, according to IAPT, exchange rates play a crucial role in converting the foreign prices of risk into domestic ones. The latter hypothesis can be tested by means of equation (25). Moreover, we can perform an even stronger additional test for risk conversion, explicitly including the covariances between exchange rates and local currency asset returns. These moments have been discussed in equation (19) above.

4 Empirical Results

We focus attention on a 3-country arbitrage model for hedged bond portfolio excess returns. The countries considered are the US, the UK, and Germany. We assume that it is possible to construct an arbitrage portfolio from these bond return portfolios, a portfolio that is locally riskfree and requires no initial investment.

4.1 Data construction and description

4.1.1 Portfolio returns (in local currency)

We construct (LIBOR and swap) yields using the yields as implied by LIBOR and swap rates (Piazzesi (2001, p. 18, equation 12)). Monthly observed LIBOR rates of 1 to 12 month maturities are retrieved for the US, UK and Germany. Swap rates are retrieved as well for maturities from 2 to 10 years. We picked every first Tuesday of each month from Datastream. In the end, the maturity spectrum that we construct consists of 18 yield time series with time to maturities of 1, 2, ..., 12 months, and 2, 3, 4, 5, 7 and 10 years. We use 162 monthly observations on zero coupon bond yields and relevant exchange rates for the US, the UK, and Germany. The first data point is end of April 1987 (29/04/1987), and the last is end of September 2000 (27/09/2000). From these, 162 bond prices and $T = 161$ monthly bond (holding) returns are constructed. Annualized analogues for means and standard deviations are obtained by multiplying their monthly counterparts by 12 and $\sqrt{12}$ respectively. From the original 18 maturity return spectrum that is available to us, we construct 4 portfolio return series to decrease the idiosyncratic risk in every single bond return. The 4 return portfolios can be regarded to be a short, short-mid, long-mid, and long-maturity spectrum return portfolio. The first portfolio is constructed by combining the 3, 6 and 9month bond returns, the second by combining the 1, 2, 3 year, the third by combining the 4 and 5 year, and finally the last by combining the 7 and 10 year maturities, all with equal weights.

In the end, we obtain $N = 4$ bond portfolio return time series for each of the $M = 3$ countries considered. We label all relevant variables with *us*, *uk*, and *ger* for asset origin, and with *usd*, *gbp*, and *dem* for currency denomination. We group this return portfolio observations in 12 time series vectors r_{us_i}, r_{uk_i} and r_{ger_i}, $i = \{1, ..., 4\}$. We also retain but do not report the one month bond return series $r_{us_{1m}}$, $r_{uk_{1m}}$, and $r_{ger_{1m}}$which we used to construct excess returns $\tilde{r}_{us_i}$, $\tilde{r}_{uk_i}$ and $\tilde{r}_{ger_i}$, $i = \{1, ..., 4\}$. The base currency of this original dataset is the local currency. Some summary statistics for this dataset may be retrieved in **table 1.**

Insert table 1 here

We see that within each country, the mean of the portfolio returns is upward sloping. The same monotonicity holds for the standard deviation of the portfolio returns. Normality is rejected for most series and both excess kurtosis and skewness are underlying causes for this. Finally, monthly autocorrelations are fairly high for the short term portfolio, decreasing fast towards the longer term portfolios.

4.1.2 Common currency returns (unhedged and hedged)

The returns in table 1 can not be used to evaluate the strategy of a typical internationally oriented investor. Indeed, at the end of the day, investors do not care about returns in foreign currency since they face exchange rate changes when evaluating the proceeds of their international investment strategy. We first have to transform the foreign currency returns into returns expressed in the investor's home currency. These common currency excess holding returns can be easily computed by means of the discrete time equivalent of equation (3) for unhedged portfolios and (13) for hedged returns. **Tables 2** and **3** present some summary statistics on the holding returns for the alternative portfolios stated in the different currency bases.

Insert tables 2 and 3 here

A first observation to be made is the major difference between hedged and non-hedged common currency excess returns. In particular, it can be seen that hedging provides a great deal of variance reduction. The reduction in variance compared to the unhedged returns amounts to about 75-85%, corroborating the fact that exchange rate changes form the primary source of risk in international unhedged bond portfolios. The results of a variance decomposition (not reported here, available on request) confirm that the observed increased variability of common currency returns indeed originates from the variability of the exchange rate (see also Annaert (1994) for example). Second, note that the reduction in risk, as measured by the variance is *not* accompanied by an equivalent reduction in the mean return. In fact, in our sample, some of the mean returns of hedged portfolios exceed the ones for non-hedged returns. This can be attributed to the mean depreciation rate of the exchange rate. Third, and most important in this research is the fact that the mean returns of the hedged portfolios do *not* coincide with the local currency returns, notwithstanding the fact that exchange rate risk is hedged. The fact that these mean returns do not coincide is a key indication that the exchange rate plays a fundamental role, not only in the determination of the volatility of the portfolio but also in the determination of the mean return. The latter effect is often ignored in the literature because of its relative small size.

Insert table 4 here

Even though the asset return/exchange rate return cross-term may be small in absolute terms it cannot be ignored for two reasons. First, it provides a key indication of the role of the exchange rate in international finance. More specifically, this cross-term theoretically ensures that foreign currency prices of risk are converted into local prices of risk. Second, even though the effect of the cross term may be small in absolute size, it is often *statistically* significant, indicating some structural effect of the cross-term on the mean holding returns. In **table 4** we present the correlation coefficients of the local excess holding returns with the exchange rate returns. As can be seen from this table, correlation coefficients are often significant at the 95% confidence level.

4.2 Practical implementation

As mentioned before, in the practical implementation of the test statistics we use a two step approach. The first step consists of a standard factor analysis on local currency excess returns to filter the international factors $\mathbf{F}_t$ and their respective $NM \times K$ loading matrix $\mathbf{L} = (\mathbf{L}_1, ..., \mathbf{L}_K)$. The second step then proceeds with a generalized method of moment approach (Hansen (1982)) to estimate the country-specific market prices of risk and to test the overidentifying restrictions of the model. To extract the international factors and their loadings from local currency bond returns we follow the methodology as set out in Ahn (1997) and Driessen, Melenberg, and Nijman (2000). In line with the latter, we assume the following linear orthogonal factor model for excess bond portfolio returns:

$$\widetilde{\mathbf{r}}_t = \boldsymbol{\mu} + \mathbf{L}\mathbf{F}_t + \boldsymbol{\varepsilon}_t \tag{27}$$

where $\mathbf{F}_t$ denotes a $K \times 1$ vector of international factor risks, independent of the currency specification. Moreover, we impose an orthogonal, unit variance factor structure:

$$E(\mathbf{F}_t) = \mathbf{0}_K \qquad E(\mathbf{F}_t\mathbf{F}_t') = \mathbf{I}_K \qquad E(\mathbf{F}_t\boldsymbol{\varepsilon}_t') = \mathbf{0}_{K\times NM} \qquad E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \boldsymbol{\Psi}. \tag{28}$$

Given these assumptions, it is straightforward to show that the covariance matrix (the correlation matrix is obtained by applying this approach to rescaled excess returns) of local currency excess returns, $\boldsymbol{\Omega} \equiv \mathbf{E}(\mathbf{r}_t\mathbf{r}_t')$, may be decomposed as:

$$\boldsymbol{\Omega} = \mathbf{L}\mathbf{L}' + \boldsymbol{\Psi}. \tag{29}$$

An eigenvalue-eigenvector decomposition of the matrix $\boldsymbol{\Omega}$ allows us to recover the loading matrix $\mathbf{L}$ and the associated factors can be extracted by noting that $\hat{\mathbf{F}}_t = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{L}'(\tilde{\mathbf{r}}_t - \boldsymbol{\mu})$. As shown by Basilevsky (1994), standard procedures as discussed above need to be rescaled to avoid upward biases in the loadings. More details on the rescaling procedure can be found in **appendix A**. Note that the assumption of independence and unit variance on the factors (see equation (28)) is sufficient to identify the factors up to a sign transformation. These sign transformations will determine the sign of the prices of risk but will, however, not affect the test statistics (equations (25) and (26)). Determining the appropriate number of factors to adequately describe the correlation between bond returns largely remains a subjective issue. It is argued in the literature that a visual inspection of the residual matrix should be combined with a SCREE test and common sense to come up with the number of factors needed. Though the SCREE test suggests a minimal number of three factors to be adequate, we found the entries in the residual matrix too high and positively affected by adding a fourth factor. In the following, we choose to report results with factor numbers ranging from $K = 1$ until $K = 11$. Obviously, not all these settings are equally plausible.

4.3 Three-country estimation results

4.3.1 Test results

In this section, we present evidence that prices of risk are different across countries and that the model cannot be rejected for some (though not all) model specifications. The latter implies that the exchange rate effectively converts prices of international risks, as argued above in the theoretical section. We performed a GMM overidentifying restrictions test on a battery of different model specifications. The model specifications differ with respect to *(i)* the number of factors used for the linear orthogonal factor model, *(ii)* the number of overidentifying restrictions (moments), and *(iii)* the number of free parameters allowed for.

Country-specific market prices of risk estimates The estimated currency-specific prices of risk for the case where 11 factors are retained may be retrieved in **table 5**. The prices of risk do not differ markedly between models for different K, due to the orthogonality of the K factors (not shown in table). So the above results for the 11 factor version encapsulate the results of lower factor numbers as well.

Insert table 5 here

The fact that we used the correlation matrix of the data as the basis for the spectral decomposition will of course have an effect on the size of the prices of risk and the factor loadings (prices of risk will be higher and loadings lower when using the covariance matrix). However, in the end the same result would emerge in terms of expected excess returns (see expected bond return illustration below).

Are prices of risk different across countries? Before testing if there is evidence for price of risk conversion across countries, one should ascertain that the prices of risk are indeed statistically different from each other across countries. In **panel A** of **table 6** we report $\chi^2_{NM^2-K}$ statistics and corresponding p-values when using ($NM^2 = 36$) excess return moments based upon equation (26) and where we have imposed prices of risk to be equal across countries, $\boldsymbol{\lambda}^{usd} = \boldsymbol{\lambda}^{gbp} = \boldsymbol{\lambda}^{dem}$. Imposing equality of prices of risk across currencies, we clearly reject the null hypothesis (the validity of the 3-country IAPM) in terms of any number of factors that span local currency bond returns. Indeed, p-values corresponding to χ^2_{36-K} statistics do not exceed 5% for any reasonable number of factors used in the first column.

Insert table 6 here

Do exchange rates equalize prices of risk across currencies? The 3-country model test results when loosening the assumption of equal prices of risk across countries are presented in panel B. The **first column** of **panel B** uses exactly the same moments as in panel A and thus presents p-values between brackets that correspond to $\chi^2_{NM^2-3K}$ statistics. We see now that after allowing the prices of risk to differ between countries we are no longer able to reject the model (at 95% confidence interval) when 6 or more international factors are allowed for. These results imply the validity of the model and of the exchange rate being a price of risk converter across currencies.

To strengthen the model test we explicitly include all covariance moments between exchange rate returns and local currency returns as extra overidentifying moments, making the test even stronger. We construct the $(M-1)NM = 24$ moments that correspond to equation (14) above. The **second column** in **panel B** reports χ^2_{60-3K} test statistics where all $2NM^2 - NM = 60$ moments, as defined in equation (14) and (19), are included. We see that, when taking all overidentifying moments into consideration, the evidence in favor of the model does not disappear. In a range between 4 and 7 factors, the model cannot be rejected. The test statistics of panel B are the main findings of this paper and indeed suggests that the exchange rate converts market prices of risk from one currency to another.

4.3.2 Illustration: is risk conversion present in expected returns?

If our model cannot be rejected, risk conversion should be present in expected returns. To visualize this, we compare excess returns from equivalent investments in different currencies. Assume a *us* investor who considers a *uk* investment ($usd = m$ and $gbp = m^*$). The expected excess hedged return of this investment can be retrieved in equation (19), adapted to our setting :

$$\begin{aligned} \mathbf{L}_n^{gbp}\boldsymbol{\Gamma}_{gbp}^{usd} &= \mathbf{L}_n^{gbp}\left(\boldsymbol{\lambda}^{usd} - \boldsymbol{\lambda}^{gbp}\right), \\ \mathbf{L}_n^{gbp}\boldsymbol{\lambda}^{usd} &= \mathbf{L}_n^{gbp}\boldsymbol{\lambda}^{gbp} + \mathbf{L}_n^{gbp}\boldsymbol{\Gamma}_{gbp}^{usd}, \end{aligned} \tag{30}$$

where $\mathbf{L}_n^m = (L_{n1}^m, ..., L_{nK}^m)$, and $\boldsymbol{\Gamma}_{gbp}^{usd} = \left(\Gamma_{gbp,1}^{usd}, ..., \Gamma_{gbp,K}^{usd}\right)'$ are $1 \times K$ and $K \times 1$ vectors respectively. According to IAPT, the expected return that the *us* investor expects to receive on his *uk* investment is determined by the *us* prices of risk. The right hand side shows that the (possibly favorable) foreign prices of risk can not be reaped and are converted to the domestic ones by correcting for exchange rate risk. In practice, we obtain estimates $\hat{\boldsymbol{\Gamma}}_{gbp}^{usd}$ of the above theoretical exchange rate loadings $\mathbf{\Gamma}_{gbp}^{usd}$ from regressing unexpected exchange rate changes upon the K filtered factors (see **table 7**)[5].

Insert table 7 here

Because of no-arbitrage conditions, the observed average excess return of *gbp* investments after exchange rate conversion to *usd* should not deviate statistically from the average excess returns expected on the equivalent domestic *usd* portfolio.

$$\mathbf{L}_n^{gbp}\boldsymbol{\lambda}^{usd} \approx \mathbf{L}_n^{gbp}\boldsymbol{\lambda}^{gbp} + \mathbf{L}_n^{gbp}\hat{\mathbf{\Gamma}}_{gbp}^{usd}, \tag{31}$$

The difference between the left and right hand side is due to the estimated difference between the regression loadings for the exchange rate $\hat{\mathbf{\Gamma}}_{gbp}^{usd}$ and the arbitrage imposed population value, *i.e.* $\mathbf{\Gamma}_{gbp}^{usd} = \boldsymbol{\lambda}^{usd} - \boldsymbol{\lambda}^{gbp}$. In returns, this difference is due to imperfect risk conversion:

$$\text{size of imperfect risk conversion} \ = \ \mathbf{L}_n^{gbp}(\boldsymbol{\lambda}^{usd} - \boldsymbol{\lambda}^{gbp} - \hat{\mathbf{\Gamma}}_{gbp}^{usd}). \tag{32}$$

In **figure 1** we visualize expected annualized excess returns in local currency and in common currency. The horizontal axis of the top/middle/lower two panels give the expected excess return on the *us/uk/ger* bond portfolios as received by their domestic investor (respectively *us/uk/ger*). The vertical axis denotes the expected excess return of the foreign investors before (black full dots) and after (empty squares) common currency denomination. The plain line is the 45 degrees line and denotes perfect risk conversion or equality of returns. Visually and in most cases, the risk conversion effect of the exchange rate indeed brings expected returns closer to the ones of the domestic investor (closer to the 45 degree line). Or, after correcting for the exchange risk, expected returns are more equal. The lower right panel seems to be the exception and corresponds to a German investor that invests in the UK. One reason for the absence of perfect conversion, *i.e.* for the remaining difference in excess returns, can obviously be that the OLS based exchange rate sensitivities contain estimation errors. To take this into account we added 95% confidence intervals around the hypothesized value $\mathbf{\Gamma}_{gbp}^{usd} = \boldsymbol{\lambda}^{usd} - \boldsymbol{\lambda}^{gbp}$. As can be inferred from the figure, all estimated deviations after exchange rate conversion fall within these bounds, yielding the conclusion that we cannot reject the hypothesis of appropriate exchange rate conversion in a statistical sense at the 5% level. This evidence illustrates and suggests again that exchange rates do convert foreign prices of risk to the domestic ones.

Insert table 8 here

[5] **Table 7** reports OLS coefficients when exchange rate returns (in logs) are regressed upon the international factors that span local currency bond returns. Results are reported for the maximum number of factors considered (*i.e.* 11). Note that results for $K < 11$ are just a subset of the table, of course, due to the assured orthogonality of the factors. From the low coefficients of determination (not reported here, available on request), it appears that the exchange rate adds some new risk that cannot be captured by bond market returns. The R^2 ranges from a low of 1.5% for the gbp/dem exchange rate return (3 bond return factors) to a high of 26.5% (11 factors, S_{gbp}^{usd} return). Typically, it is about 10% to 15%. Apparently, currency fluctuations are largely explained by non-bond market movements (stock price returns, macroeconomic factors and idiosyncratic noise).

In order to examine whether or not this bias is within the limits of transaction costs, we computed the absolute *per annum* difference between the domestic expected excess return and the one that a foreign investor receives, before and after risk conversion. The results are presented in **table 8**. From the table, it can be seen that the risk conversion indeed reduces the difference between expected excess returns between domestic and foreign investors. This is true on average for all cases, except for the German bonds invested in by *uk* investors. In any case, the increase in the difference is not significant enough to present statistical evidence against the model, as we have seen in table 6. In all other cases, this difference is reduced, sometimes markedly (e.g. for a *uk* investor that invests in the *us* the difference drops from an average of 4.52% to 0.53%, well within transaction cost limits). If we assume average transaction costs to be as high as 2.5%, then the risk reduction can be considered adequate. If transaction costs are esteemed to be lower on average, the differentials could be due to a lack of data and hence significance of point estimates.

5 Conclusions

The simultaneous and arbitrage-free modeling of asset returns and exchange rates yields several testable model claims. The most important implication is that the exchange rate serves the role of equalizing prices of risk across currencies. Brandt and Santa-Clara (2001), Bansal (1995), Lund (1999) and Dewachter and Maes (2001) all derive and discuss this key relationship in a conditional framework. Of course, if this relation is to hold conditionally, then it should at least hold unconditionally as well (the opposite is not necessarily true). Hence, in this paper we focus on the unconditional version of this no-arbitrage corollary, using a linear factor model for local currency bond returns.

We employ a two step approach to estimate the model for US, UK, and German bond portfolio return data. The first step consists of a standard factor analysis on local currency excess returns to filter the international factors and their respective loading matrix. The second step then proceeds with a generalized method of moment approach to estimate the country-specific market prices of risk. The latter step enables us to test many of the model claims by means of simple overidentifying restrictions tests.

Our main findings may be summarized as follows. First, we find statistical evidence that prices of risk differ across countries. Second, employing overidentifying restrictions tests for a battery of model specifications we are unable to reject the hypothesis that the exchange rate is an unconditional risk price converter across currencies. This conclusion is robust to the explicit inclusion of the covariance restrictions between asset price returns and exchange rate returns. Third and as an illustration, we show that expected excess returns from domestic and foreign investments (with equal loadings) approach one another in a statistically and visibly significant way when you convert the foreign currency investment proceeds into a common currency. Though the difference in expected excess returns does not disappear completely, the absolute size of the difference seems to be within transaction cost limits. Overall, results call for more unconditional and conditional research on the role of exchange rates within arbitrage-free pricing models.

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Appendix: principal factor analysis

An orthogonal factor model with K common factors is summarized as follows:

$$\underset{(NM\times 1)}{\tilde{\mathbf{r}}_t - \boldsymbol{\mu}} = \underset{(NM\times K)}{\mathbf{L}}\underset{(K\times 1)}{\mathbf{F}_t} + \underset{(NM\times 1)}{\boldsymbol{\varepsilon}_t} \tag{33}$$

with:

$$\begin{array}{rclrl} E(\tilde{\mathbf{r}}_t) & = & \boldsymbol{\mu} & E(\tilde{\mathbf{r}}_t\tilde{\mathbf{r}}_t') & = \boldsymbol{\Omega} \\ E(\mathbf{F}_t) & = & \mathbf{0} & E(\mathbf{F}_t\mathbf{F}_t') & = \mathbf{I}_K \\ E(\boldsymbol{\varepsilon}_t) & = & \mathbf{0} & E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') & = \boldsymbol{\Psi} \end{array} \tag{34}$$

where $\boldsymbol{\Psi}$ is diagonal. This orthogonal model implies following covariance structure for $(\tilde{\mathbf{r}}_t - \boldsymbol{\mu})$:

$$\boldsymbol{\Omega} = \mathbf{L}\mathbf{L}' + \boldsymbol{\Psi} \tag{35}$$

To estimate the elements $\mathbf{L}$ the method of principal component analysis is widely used. Spectral decomposition allows us to express the correlation matrix $\boldsymbol{\Omega}$ as:

$$\boldsymbol{\Omega} = \Lambda_{1,1}\mathbf{e}_1\mathbf{e}_1' + ... + \Lambda_{NM,NM}\mathbf{e}_{NM}\mathbf{e}_{NM}' \tag{36}$$

where $\Lambda_{1,1} > ... > \Lambda_{NM,NM}$ are the ordered eigenvalues and $\mathbf{e}_1$, $\mathbf{e}_2$, ..., $\mathbf{e}_{NM}$ are the associated normalized eigenvectors such that $\mathbf{e}_i\mathbf{e}_i' = 1$. As we want to explain the covariance structure in terms of $K < NM$ factors we reduce the above decomposition to:

$$\begin{aligned} \mathbf{W} &= (\Lambda_{1,1}\mathbf{e}_1\mathbf{e}_1' + ... + \Lambda_{K,K}\mathbf{e}_K\mathbf{e}_K') + (\Lambda_{K+1,K+1}\mathbf{e}_{K+1}\mathbf{e}_{K+1}' + ... + \Lambda_{NM,NM}\mathbf{e}_{NM}\mathbf{e}_{NM}') \\ &= \begin{bmatrix} \sqrt{\Lambda_{1,1}}\mathbf{e}_1 & ... & \sqrt{\Lambda_{K,K}}\mathbf{e}_K \end{bmatrix} \cdot \begin{bmatrix} \sqrt{\Lambda_{1,1}}\mathbf{e}_1' \\ \vdots \\ \sqrt{\Lambda_{K,K}}\mathbf{e}_K' \end{bmatrix} + \begin{bmatrix} \psi_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \psi_{NM} \end{bmatrix} \\ &= \mathbf{L}\mathbf{L}' + \boldsymbol{\Psi} \end{aligned} \tag{37}$$

where it is assumed that $\boldsymbol{\Psi} = \mathbf{W} - \mathbf{L}\mathbf{L}'$ is obtained by setting the off-diagonal elements to zero.[6] This methodology to obtain estimates of $\mathbf{L}$ and $\boldsymbol{\Psi}$, applied to the sample correlation matrix $\mathbf{S}$ is known as the ordinary principal component analysis, as the factor loadings are in fact scaled coefficients of the first K principal components. Basilevsky (1994) shows that the ordinary principal component solution is upward biased. Therefore one has to multiply the solution by a scale factor $\sqrt{\dfrac{\Lambda_{j,j} - \sigma^2}{\Lambda_{j,j}}}$. When the residual variance turns out to be small, these scale factors will only be slightly smaller than one. The complete solution for the factor loadings is then:

$$\mathbf{L} = \begin{bmatrix} \sqrt{\Lambda_{1,1}}\mathbf{e}_1 \cdot \sqrt{\dfrac{\Lambda_{1,1} - \sigma^2}{\Lambda_{1,1}}} & ... & \sqrt{\Lambda_{K,K}}\mathbf{e}_K \cdot \sqrt{\dfrac{\Lambda_{K,K} - \sigma^2}{\Lambda_{K,K}}} \end{bmatrix} \tag{38}$$

[6]When the off-diagonal elements are not close to zero, this may indicate ommited factors.

Now that we have defined factor loadings $\mathbf{L}$ and the variance of the specific components $\mathbf{\Psi}$ we can generate estimates of the values for the latent factors $\mathbf{F}_t$ using a simple OLS procedure (It is assumed that $\mathbf{\Psi}$ is homoskedastic):

$$\begin{aligned}
\mathbf{F}_t &= \left(\mathbf{L}'\mathbf{L}\right)^{-1}\mathbf{L}'\left(\tilde{\mathbf{r}}_t - \boldsymbol{\mu}\right) \\
&= \begin{bmatrix} \frac{1}{\sqrt{\Lambda_{1,1}}}\mathbf{e}_1'\left(\tilde{\mathbf{r}}_t - \boldsymbol{\mu}\right) \\ \vdots \\ \frac{1}{\sqrt{\Lambda_{K,K}}}\mathbf{e}_K'\left(\tilde{\mathbf{r}}_t - \boldsymbol{\mu}\right) \end{bmatrix}
\end{aligned} \tag{39}$$

Tables and Figures

Table 1: **Local currency portfolio return summary statistics (1987:4-2000:9).**

Portfolio	**Mean**	**Std**	**Min**	**Max**	**Kurt**	**Skew**	**JB**	$\boldsymbol{\rho}_1$
us_1	0.0644	0.0067	0.0139	0.1425	3.7620	0.6886*	16.6204*	0.6292
us_2	0.0759	0.0237	-0.1232	0.2861	2.6484	0.1557	1.4804	0.2136
us_3	0.0889	0.0503	-0.3540	0.5429	2.7898	0.0615	0.3979	0.1340
us_4	0.1083	0.0876	-0.7428	0.9280	3.0504	0.0372	0.0542	0.0515
uk_1	0.0887	0.0109	-0.0070	0.2149	3.4823	0.9189*	24.2169*	0.6733
uk_2	0.0947	0.0272	-0.1849	0.5466	7.3402*	1.1829*	163.9106*	0.2526
uk_3	0.1033	0.0548	-0.5757	0.8618	6.0077*	0.6719*	72.8005*	0.1963
uk_4	0.1227	0.0895	-0.9928	1.0556	4.0922*	0.1486	8.5947*	0.1537
ger_1	0.0568	0.0077	0.0040	0.1458	2.5862	0.5822*	10.2449*	0.7836
ger_2	0.0616	0.0195	-0.1198	0.3629	5.1004*	0.5148*	36.7055*	0.3877
ger_3	0.0706	0.0400	-0.3456	0.5478	3.7467	-0.0123	3.7446	0.2879
ger_4	0.0840	0.0634	-0.7849	0.5971	3.6496	-0.4680	8.7079*	0.1873

Entries concern return summary statistics. *Mean* denotes the arithmetic average, *Std* standard deviation, *Min* and *Max* minimum and maximum, *Kurt* excess kurtosis, *Skew* skewness, *JB* Jarque-Bera test statistic for normality and ρ_1 first order autocorrelation. Statistical significance at the 5% level for *Kurt*, *Skew* and *JB* is denoted by a superscript asterisk. Critical values are ± 0.512 for Skew, 1.967 and 4.024 for Kurt, 5.991 for JB. Mean, Min and Max are annualized by multiplying by 12. Standard deviation by multiplication of $\sqrt{12}$. All statistics (and critical values) are based upon 161 observations.

Table 2: **Common currency unhedged portfolio return selected summary statistics (1987:4-2000:9).**

Portfolio	**usd**		**gbp**		**dem**	
	Mean	**Std**	**Mean**	**Std**	**Mean**	**Std**
us_1	0.0644	0.0067	0.0795	0.1070	0.0863	0.1079
us_2	0.0759	0.0237	0.0907	0.1063	0.0972	0.1055
us_3	0.0889	0.0503	0.1035	0.1134	0.1097	0.1100
us_4	0.1083	0.0876	0.1230	0.1351	0.1289	0.1291
uk_1	0.0846	0.1058	0.0887	0.0109	0.0981	0.0756
uk_2	0.0902	0.1057	0.0947	0.0272	0.1040	0.0781
uk_3	0.0984	0.1126	0.1033	0.0548	0.1126	0.0919
uk_4	0.1177	0.1322	0.1227	0.0895	0.1324	0.1194
ger_1	0.0464	0.1086	0.0533	0.0798	0.0568	0.0077
ger_2	0.0512	0.1100	0.0582	0.0841	0.0616	0.0195
ger_3	0.0601	0.1139	0.0673	0.0920	0.0706	0.0400
ger_4	0.0734	0.1243	0.0806	0.1041	0.0840	0.0634

Entries concern return summary statistics. The common currencies in which the returns are expressed may be retrieved in the first row. *Mean* denotes the arithmetic average, *Std* standard deviation. The mean is annualized by multiplying by 12. Standard deviation by multiplication of $\sqrt{12}$. All statistics (and critical values) are based upon 161 observations.

Table 3: **Common currency hedged portfolio return summary statistics (1987:4-2000:9).**

Entries concern return summary statistics. *Mean* denotes 12 monthly mean, *Std* $\sqrt{12}$ times the monthly standard deviation, *% RR* the % risk reduction, as approximated by the % reduction in portfolio return variance compared to the unhedged alternative. The average of these is taken over all (nonzero) elements. Note that we computed hedged returns according to the approximation as presented in Annaert (1994). The exact formula is only slightly more complicated and yielded almost identical results.

Portfolio	**usd**			**gbp**			**dem**		
	Mean	**Std**	**% RR**	**Mean**	**Std**	**% RR**	**Mean**	**Std**	**% RR**
us_1	0.0644	0.0067		0.0383	0.0084	-99.39	0.0691	0.0128	-98.58
us_2	0.0759	0.0237		0.0498	0.0236	-95.06	0.0806	0.0255	-94.14
us_3	0.0889	0.0503		0.0628	0.0500	-80.59	0.0936	0.0507	-78.74
us_4	0.1083	0.0876		0.0823	0.0873	-58.19	0.1130	0.0877	-53.82
uk_1	0.1148	0.0167	-97.52	0.0887	0.0109		0.1195	0.0161	-95.46
uk_2	0.1208	0.0304	-91.71	0.0947	0.0272		0.1255	0.0290	-86.20
uk_3	0.1293	0.0568	-74.54	0.1033	0.0548		0.1341	0.0553	-63.79
uk_4	0.1488	0.0909	-52.66	0.1227	0.0895		0.1535	0.0896	-43.66
ger_1	0.0520	0.0152	-98.03	0.0260	0.0115	-97.92	0.0568	0.0077	
ger_2	0.0569	0.0242	-95.15	0.0308	0.0222	-93.04	0.0616	0.0195	
ger_3	0.0659	0.0429	-85.82	0.0398	0.0420	-79.17	0.0706	0.0400	
ger_4	0.0792	0.0653	-72.41	0.0532	0.0648	-61.28	0.0840	0.0634	
AVER			-83.48			-83.08			-76.80

Table 4: **Correlations between local currency returns and exchange rate returns (1987:4-2000:9).**

	usd/gbp	**usd/dem**	**gbp/dem**
us_1	0.1004 (1.2721)	0.1247 (1.5850*)	
us_2	0.1656 (2.1173*)	0.2438 (3.1703*)	
us_3	0.1218 (1.5474*)	0.2133 (2.7534*)	
us_4	0.0627 (0.7922)	0.1581 (2.0186*)	
uk_1	-0.0338 (-0.4267)		0.1805 (2.3137*)
uk_2	-0.1459 (-1.8594*)		0.1369 (1.7426*)
uk_3	-0.1461 (-1.8627*)		0.0610 (0.7707)
uk_4	-0.1043 (-1.3227)		-0.0196 (-0.2476)
ger_1		0.1380 (1.7564*)	0.2169 (2.8013*)
ger_2		0.0408 (0.5146)	0.1913 (2.4582*)
ger_3		-0.0290 (-0.3657)	0.1058 (1.3422)
ger_4		-0.0221 (-0.2788)	0.0586 (0.7403)

Entries are correlations between local currency returns and exchange rate returns (t-statistics between brackets). The latter are computed as the first difference of the log of the level. Statistical significance at the 95% confidence level implies a cut-off point of 1.645 for a one-sided and 1.96 for a two-sided test. The correlations denominated with a superscript star are significant at the 95% confidence interval (one sided hypothesis).

Table 5: **GMM results: estimated prices of risk for the 3-country model.**

We present results for a $K = 11$ factor model. Each country is allowed to have a different price of risk on each factor, and hence we rapport the 33 market prices of risk obtained from a GMM analysis with 60 moments as specified in equation (14) and (19). Principal factor analysis was performed on the correlation matrix (see text).

k ($K = 11$)	λ_k^{usd}	λ_k^{gbp}	λ_k^{dem}
1	0.0025 (0.0019)	0.0019 (0.0021)	0.0016 (0.0013)
2	-0.0029 (0.0028)	0.0019 (0.0025)	0.0013 (0.0023)
3	0.0026 (0.0025)	0.0045 (0.0024)	0.0042 (0.0028)
4	0.0008 (0.0022)	0.0033 (0.0021)	0.0048 (0.0022)
5	0.0013 (0.0024)	0.0019 (0.0022)	-0.0006 (0.0023)
6	-0.0048 (0.0024)	-0.0028 (0.0023)	-0.0013 (0.0024)
7	-0.0058 (0.0027)	-0.0032 (0.0025)	-0.0063 (0.0026)
8	0.0034 (0.0025)	0.0000 (0.0026)	0.0022 (0.0025)
9	-0.0011 (0.0037)	0.0072 (0.0037)	0.0063 (0.0036)
10	-0.0030 (0.0020)	-0.0008 (0.0016)	-0.0020 (0.0016)
11	0.0021 (0.0024)	0.0021 (0.0029)	0.0046 (0.0028)

Table 6: **GMM overidentifying restrictions 3-country model test statistics**

The null hypothesis for this overidentifying restrictions test is the hypothesis that the 3-country IAPM is valid. Entries are χ^2_{df} -distributed statistics from a standard GMM overidentifying restrictions test, with degrees of freedom (df) equal to the number of moments minus the number of free parameters (prices of risk). Between brackets are the corresponding p-values in % points. **Panel A** shows the model test statistics TJ^2_T of equation (26) where we imposed the market prices of risk to be equal across countries (#free parameters = K) and where moments (14) are used. **Panel B** allows prices of risk to differ between countries (# free parameters = $3K$) and performs the same test. The first column corresponds to TJ^1_T of equation (25). The second column of panel B presents results where the covariance moments of equation (19) are included as well.

	PANEL A	**PANEL B**	
	$\boldsymbol{\lambda}^{usd} = \boldsymbol{\lambda}^{gbp} = \boldsymbol{\lambda}^{dem}$	$\boldsymbol{\lambda}^{usd} \neq \boldsymbol{\lambda}^{gbp} \neq \boldsymbol{\lambda}^{dem}$	
K	χ^2 (p-value in %)	χ^2 (p-$value$ in %)	
	$\chi^2_{NM^2-K}$	$\chi^2_{NM^2-3K}$	$\chi^2_{2NM^2-NM-3K}$
1	65.3 (0.1)	33.0 (0.2)	57.0 (0.0)
2	63.9 (0.1)	30.0 (0.5)	54.0 (0.0)
3	60.6 (0.2)	27.0 (1.7)	51.0 (0.0)
4	60.3 (0.2)	24.0 (1.3)	48.0* (12.5)
5	60.3 (0.1)	21.0 (2.1)	45.0* (11.4)
6	51.0 (1.0)	18.0* (8.1)	42.0* (13.6)
7	51.0 (0.7)	15.0* (9.3)	39.0* (7.8)
8	50.9 (0.5)	12.0* (7.1)	36.0 (4.1)
9	50.6 (0.4)	9.0* (36.1)	33.0 (2.5)
10	49.5 (0.4)	6.0* (41.3)	30.0 (1.3)
11	48.8 (0.3)	3.0* (44.6)	27.0 (0.6)

Table 7: **OLS estimates from regressing exchange rate returns upon international local currency factors.**

The table reports OLS coefficients from the following regression: $\Delta S^{hc}_{fc} = \Gamma^{hc}_{fc,0} + \Gamma^{hc}_{fc,1}F_1 + ... + \Gamma^{hc}_{fc,K}F_K + \nu^{hc}_{fc}$, where ΔS^{hc}_{fc} is defined as the discrete time first difference of the corresponding log exchange rate level. The common factors F_k, $k = \{1, ..., K\}$, have been extracted from a principal factor analysis performed on local currency excess bond portfolio returns as described in the text. Standard errors are included between brackets. A superscript star denotes significance at the 5% level.

$\hat{\Gamma}^{hc}_{fc,k}$ $(K = 11)$	S^{usd}_{gbp}	S^{gbp}_{dem}	S^{usd}_{dem}
$\hat{\Gamma}^{hc}_{fc,0}$	-0.0016 (0.0023)	0.0002 (0.0018)	-0.0014 (0.0024)
$\hat{\Gamma}^{hc}_{fc,1}$	-0.0014 (0.0021)	0.0024 (0.0017)	0.0010 (0.0022)
$\hat{\Gamma}^{hc}_{fc,2}$	-0.0060* (0.0021)	-0.0010 (0.0017)	-0.0070* (0.0022)
$\hat{\Gamma}^{hc}_{fc,3}$	-0.0018 (0.0021)	-0.0003 (0.0017)	-0.0021 (0.0022)
$\hat{\Gamma}^{hc}_{fc,4}$	-0.0040 (0.0021)	-0.0026 (0.0017)	-0.0066* (0.0022)
$\hat{\Gamma}^{hc}_{fc,5}$	-0.0015 (0.0021)	0.0054* (0.0017)	0.0039 (0.0022)
$\hat{\Gamma}^{hc}_{fc,6}$	-0.0032 (0.0021)	-0.0020 (0.0017)	-0.0052* (0.0022)
$\hat{\Gamma}^{hc}_{fc,7}$	-0.0049* (0.0021)	0.0043* (0.0017)	-0.0007 (0.0022)
$\hat{\Gamma}^{hc}_{fc,8}$	0.0033 (0.0021)	-0.0018 (0.0017)	0.0014 (0.0022)
$\hat{\Gamma}^{hc}_{fc,9}$	-0.0113* (0.0021)	0.0023 (0.0017)	-0.0091* (0.0022)
$\hat{\Gamma}^{hc}_{fc,10}$	-0.0052 (0.0035)	0.0008 (0.0027)	-0.0044 (0.0036)
$\hat{\Gamma}^{hc}_{fc,11}$	0.0015 (0.0021)	-0.0027 (0.0017)	-0.0012 (0.0022)

Table 8: **Is risk conversion present in expected returns ?**

This table reports the absolute deviation of the *per annum* expected excess return between investors of different nationalities before and after exchange risk conversion and according to the results obtained with a 6 factor/60 moments model. The deviations can also be seen graphically in figure 1 as the distances to the perfect conversion line (45 degrees line). Elements (1,1) and (1,2) of the table below are computed as follows (*abs* denotes absolute value): $\mathbf{L}_n^{gbp}\text{¸}^{usd} \approx \mathbf{L}_n^{gbp}\text{¸}^{gbp} + \mathbf{L}_n^{gbp}\hat{\mathbf{\Gamma}}_{gbp}^{usd}$

before: $abs\left(\mathbf{L}_n^{gbp}\text{¸}^{usd} - \mathbf{L}_n^{gbp}\text{¸}^{gbp}\right)$

after: $abs\left(\mathbf{L}_n^{gbp}\text{¸}^{usd} - \mathbf{L}_n^{gbp}\text{¸}^{gbp} + \mathbf{L}_n^{gbp}\hat{\mathbf{\Gamma}}_{gbp}^{usd}\right)$

where *before* and *after* stand for expected excess returns in local currency and converted into common currency respectively. *aver* denotes the average absolute deviation across the four bonds.

	US bonds			
	UK investor		German investor	
%	before	after	before	after
us_1	0.0444	0.0024	0.0371	0.0220
us_2	0.0484	0.0045	0.0484	0.0274
us_3	0.0458	0.0064	0.0481	0.0267
us_4	0.0422	0.0080	0.0443	0.0237
aver	0.0452	0.0053	0.0445	0.0249
	UK bonds			
	US bonds		German investor	
%	before	after	before	after
uk_1	0.0258	0.0183	0.0210	0.0147
uk_2	0.0281	0.0229	0.0097	0.0103
uk_3	0.0291	0.0238	0.0022	0.0057
uk_4	0.0252	0.0227	0.0040	0.0029
aver	0.0270	0.0219	0.0092	0.0084
	GER bonds			
	US investor		UK investor	
%	before	after	before	after
ger_1	0.0257	0.0058	0.0073	0.0166
ger_2	0.0080	0.0035	0.0039	0.0205
ger_3	0.0031	0.0089	0.0003	0.0197
ger_4	0.0071	0.0105	0.0026	0.0191
aver	0.0109	0.0072	0.0035	0.0190

Figure 1: Application: the effect of exchange rate returns on expected bond portfolio returns across currencies (6 factor model, 60 moments used).